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寄　稿

Who is Julia Sets ?

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Abstract
Recently a young French astrophysicist asked me, "Who is Julia Sets?" Thus, in answering the correct question, "What are Julia Sets?" it seemed an appropriate occasion, in the manner of D'Arcy Thompson's book On Growth and Form, since there is a need for an organizing template that can collect disconnected objects within a coherent reference frame, to apply the notion of Julia Sets collectively to galaxies, neutron star burst oscillations, and dark matter distribution.

1 Introduction

Gaston Julia (1893-1978) and Pierre Fatou (1878-1929) were two French mathematicians who studied iteration and recursive processes in the complex plane. Their work into complex dynamics led to mathematical concepts such basins of attraction, feedback loops, fractals, and the integrating Mandelbrot Set. The fractal curve of Julia Sets may be useful as an organizing template by which to collect disconnected objects such as galaxies, neutron star burst oscillations, and dark matter distribution into a coherent reference frame.

2 Historical Background

Western certainty was shaped for two thousand years by Euclidean geometry and its integer dimensions (literally "untouched", hence "whole"). Then in 1854 Georg Riemann introduced higher dimensional space geometries. Four who utilized the fourth dimension in their work were Charles Hinton, H. G. Wells, Pablo Picasso, and Albert Einstein. However, pathologies for which no uniform integral existed began appearing in the mathematical literature, e.g., Georg Cantor, Koch, Mobius, Henri Poincaré, and Sierpinski. This appearance of chaos in the castle of certainty began the study of chaos, incomplete information, or nonlinearity. One mathematician who stressed the importance of non-integer dimensions is Benoit Mandelbrot (1924-2010).

In contrast to integer dimensions, fractal geometry may best be thought of an in-between geometry that integrates the irregular by recursion and demonstrates self-similarity. Thus, having a fractured or broken contour, fractal geometry operates as if controlled by a strange attractor endlessly looping within its closed basin on fresh orbits. Such irregular but regular shapes are far more common than basic 3D shapes.

3 Connections

The Julia Sets that interest us are the connected variety, not the Cantor Dust variety, in particular, the island variety displayed in Mandelbrot (1983, C15, Lower Archipelago), constructed by the mathematical iteration of fractal dimension 1.2.

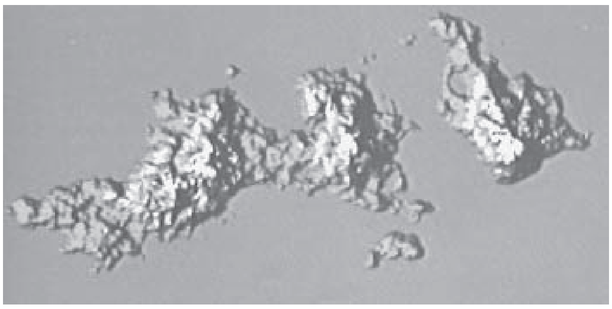

Figure 1 – Mandelbrot Island (C15)

Next, from the algorithm-generated "Mandelbrot Island", we can examine the coastlines of two real islands, Auckland Island at Google Earth, Longitude 166 degrees, 66 minutes, 34 seconds East, and Latitude 50 degrees, 42 minutes, 34 seconds South, and Great Barrier Island at Longitude 175 degrees, 25 minutes, 25 seconds, Latitude 36 Degrees, 15 minutes, fifteen seconds.

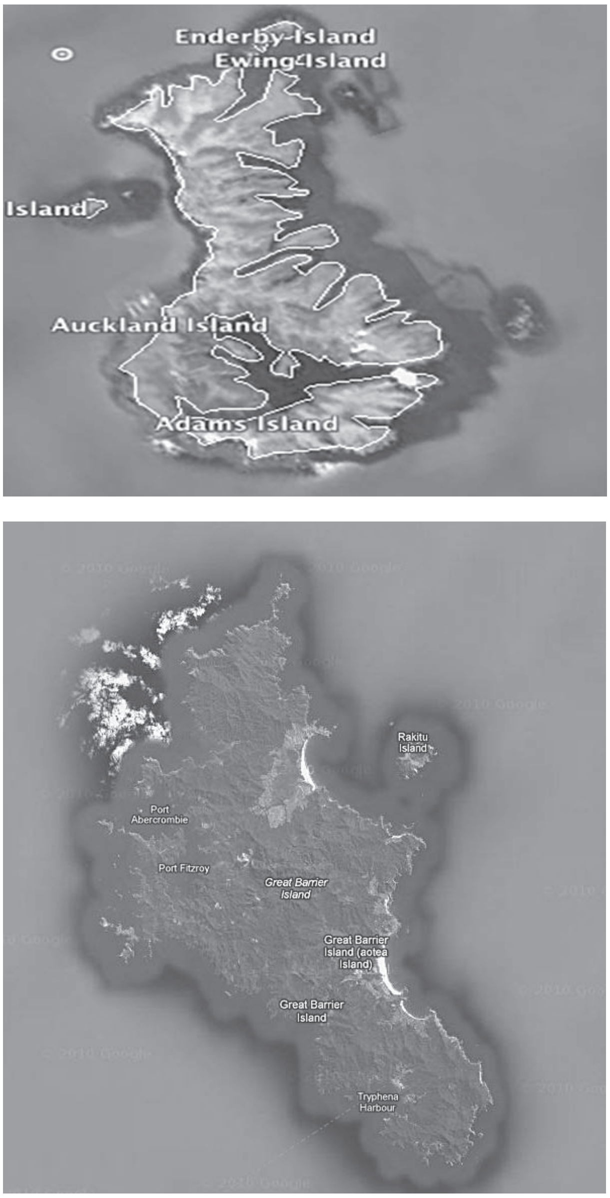

Figure 2 — Auckland Island and Great Barrier Island, New Zealand (Google Earth)

Mandelbrot Island, Auckland Island, and Great Barrier Island each appear to share the coastline fractal dimension ~1.2. Galaxies, neutron star burst oscillations, and dark matter distribution also seem to display self-similar dimensions.

4　Deforming the Line

Progressively deforming a line or a circle will bring us to the fractal dimension ~1.2.

This fractal dimensional pattern is far more common than Euclidean dimensions and, unlike Euclidean dimensions, replicates itself on all scales.

5　Galaxies

Galaxies were once classified simply as elliptical or spiral but, in the manner of an expanding Cantor Dust, growing data has led to a Galactic Zoo, or a seemingly disconnected Julia Set. The classification problem is made more difficult first by galactic alignment and second by intersecting galaxies.

Borrowing from our Mandelbrot Island, Auckland Island, and Great Barrier Island integration, we can start by identifying four main deforming (morphing) features at galaxies, i.e., Mountain, Island, Harbour, and Bay. In this progressive deformation, a mountain will deform progressively into an island; an island will deform progressively into harbours; and harbours will deform progressively into bays, first two, and then four, and then eight. But at all stages in the deformation, despite apparent change, the same fractal curve of dimension ~1.2 is being iterated at all scales.

Thus, first, one mountain (Radio Contours) equals one galaxy; two mountains equals one galaxy, three mountains equal two galaxies; and so on. Second, one island can replicate itself into an island chain. And third, harbours and bays exist, even when they are concealed by alignment and/or intersection. One handy way to think of this iteration phenomena is by isomorphism or by geometric reflection. All that remains is to track the deformations of Mountain, Island, Harbour, and Bay, which can be done by a NSEW system, or by a clock system.
Space — paragraph
"Modeling the spectral energy distribution of ULIRGs", M. S. Clemens et al., A&A 477, 95-104 (2008) DOI: 10.1051/0004-6361:200772, displays common galactic shapes that can be labeled by Mountain, Island, Harbour, and Bay.

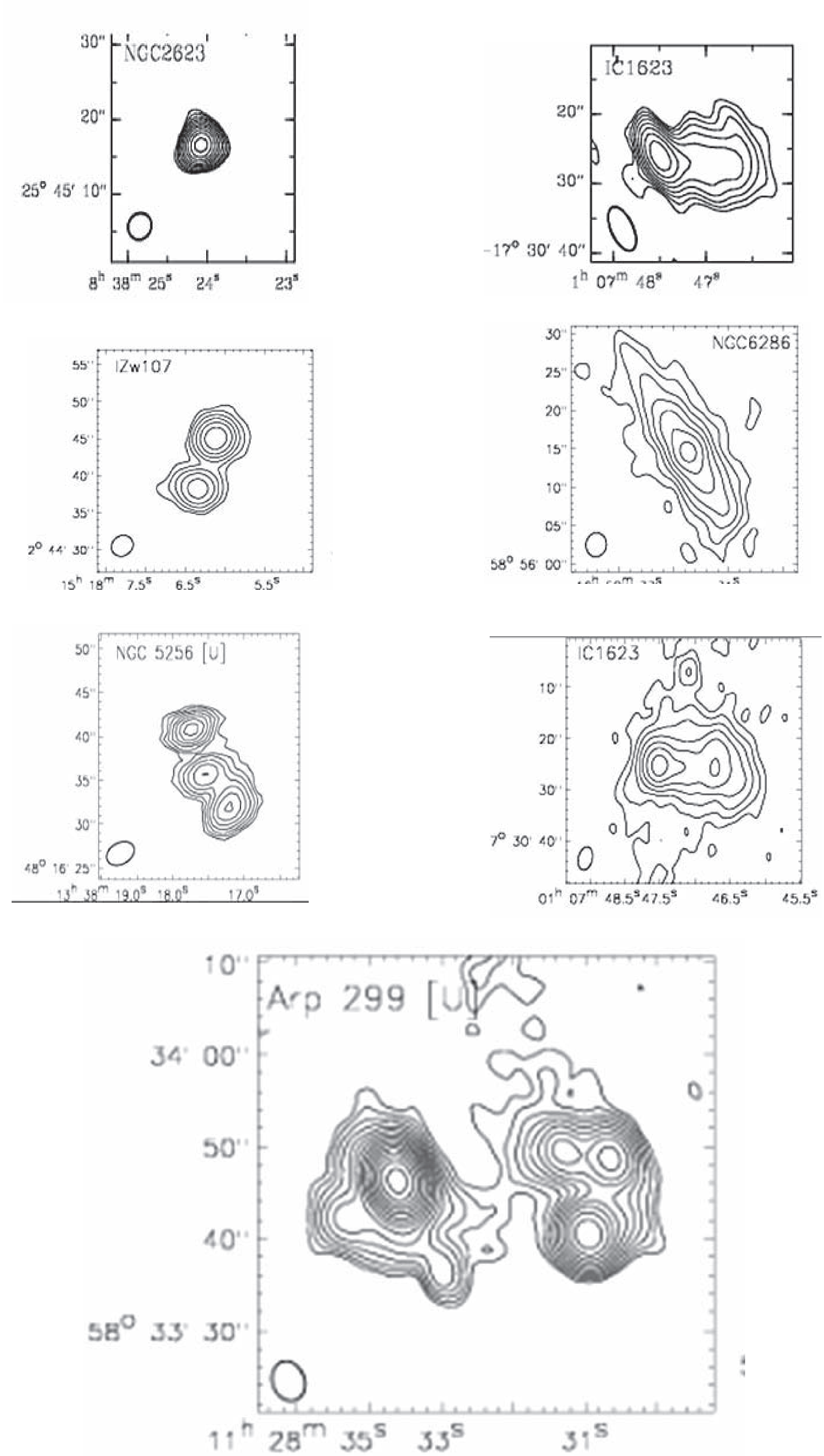

Figure 3 – Some Basic Galactic Shapes

6　Neutron Star Burst Oscillations

Neutron stars are gravitationally collapsed compact objects located this side of the event horizon of a potential black hole. Pulsars are spinning neutron stars that emit beams of electromagnetic radiation - optical, X-ray, or gamma ray - sweeping across Earth's line of sight with a regular period. These blinking pulses provide a measurement of the neutron star rotation. And millisecond pulsars are neutron stars rotating hundreds of times per second. This high speed is driven by accretion from a companion star. Currently in 2010, PSR J1748-2446ad, located in the globular cluster Terzan 5, is the fastest known pulsar spinning at 716 times a second. At its equator it spins at approximately 24% of the speed of light, or over 70,000 km per second.

Our object of interest is a thermonuclear X-ray burst from neutron star SAX J1808.4-3658 in October 2002.

Discovered in September 1998 by BeppoSAX, the accretion-powered millisecond pulsar (AMSP) SAX J1808.4-3658, is the first source to exhibit coherent X-ray pulsations and thermonuclear X-ray bursts. As the best observed member of its class, SAX J1808.4?3658 is considered to be a Rosetta Stone for low-mass X-ray binary systems (Chakrabarty & Strohmayer 1998).

Located 12,000 light years away towards the constellation Sagittarius near the direction of the galactic center at RA 18h08m27s and DEC -36d58m36s, or, in the optical at RA. = 18^h08^m2754 ± 0015 and decl. = -3658443 ± 02, or _=18:08:27.63, _=-36:58:43.37, SAX J1808.4-3658 orbits its brown dwarf companion every 2.1 hours while itself spinning at upwards of 400 times a second, giving it a spin period of 2.5 milliseconds, or a spin frequency of 401.5 Hz, SAX J1808.4-3658. Thermonuclear X-ray bursts are caused by the ignition and spreading of thermonuclear flames on the surfaces of neutron stars. The ignition material is imported from the brown dwarf companion, which is presumed to have a mass ~0.04M⊙ (assuming a fiducial NS mass of 1.4 M⊙) (Deloye et al. (2008).

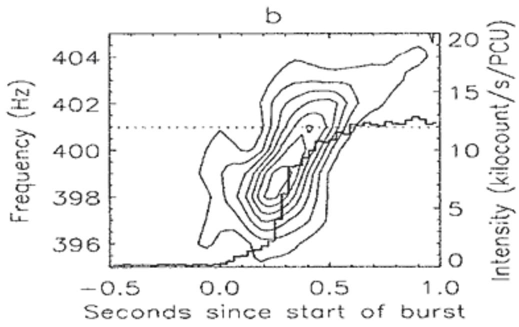

Figure 4 — October 2002 Neutron Star Burst (Chakrabarty et al. 2003)

The October 2002 neutron star burst oscillation from SAX J1808.4-3658 displays the characteristic template pattern of Mountain, Island, Harbour, and Bay, which we have associated with the formation of Julia sets and the fractal dimension 1.2.

7　Dark Matter Distribution

Figures 1, 2, 3, 4, & 5 in The Unseen Universe: Dark Matter Maps Reveal Cosmic Scaffolding, Massey et al., Nature, 18 January 2007, Letters, Pages 286-290, provides patterns that suggest dark matter follows the coastline fractal dimension ~1.2.

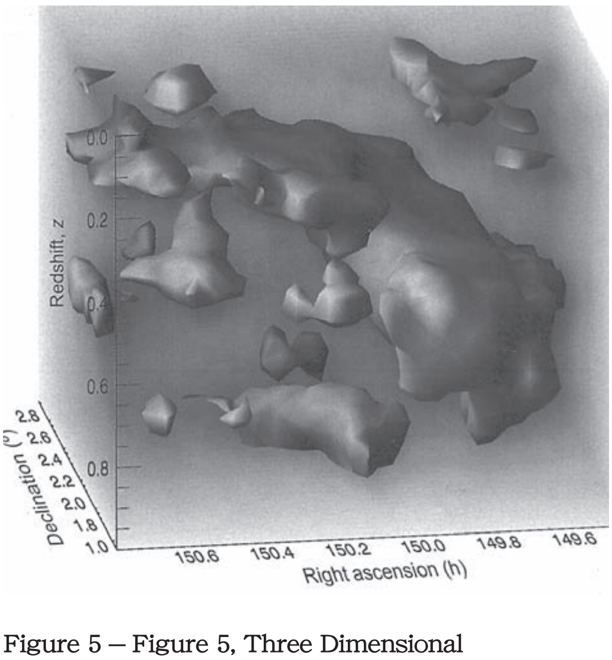

Figure 5 — Figure 5, Three Dimensional Reconstruction Dark Matter Distribution

In the distribution of dark matter, we can recover Mandelbrot Island, Auckland Island, and Great Barrier Island with each sharing a fractal dimension ~1.2. In arriving at this position, we have simply followed a deforming line or circle.

8 Conclusion

Julia Sets took us first to the coastline fractal curve of Mandelbrot's C15 Island; second to Auckland Island and Great Barrier Island, New Zealand; third, to galactic shapes; fourth, to Neutron Star bursts, and, fifth, to maps of dark matter distribution.

In our organizing template collecting disconnected objects within a coherent reference frame, we have simply been following one line, or one curve, or one circle, or one cube deforming iteratively along the fractal dimension 1.2. Our interest has been purely geometric. Thus, in answering the correct question, "What are Julia Sets?" the coastline fractal curve of 1.2 has enabled us to find similarity in diversity across a scalar range extending materially from thirty kilometres to more than 700 kpc.

Adopting a template, e.g., Great Barrier Island, could allow us to constrain galactic shapes, Neutron Star bursts, and the distribution of dark matter by one navigational system. As a consequence, the universe could be mapped by a Great Barrier Island analog, or to a deforming line, or circle, that follows fractal dimension 1.2 and displays the basins of attraction, feedback loops, and fractal iterations of a Julia Set.

9 Main Sources

Chakrabarty, D. 2003. "X-Ray Bursts & Millisecond Oscillations in the Millisecond X-Ray Pulsar SAX J1808.4-3658". http://www.sns.ias.edu/files/Chakrabarty.pdf.

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